A simple quadratic kernel for Token Jumping

Joint work with: Moritz Mühlenthaler and Daniel W. Cranston

Benjamin Peyrille

Université Grenoble Alpes, G-SCOP

January 6th 2025

Independent set reconfiguration

Let: $G = (V, E)$ be a simple graph,
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We represent vertices of I as tokens ○ and vertices of J with targets ⊕.

We want to **move** I to J iteratively, preserving the independent set property.

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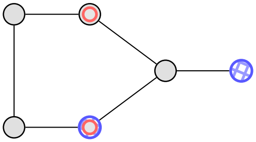

Slide along edges

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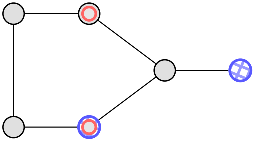

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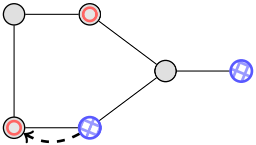

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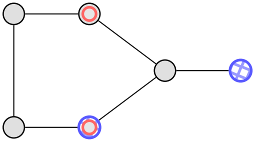

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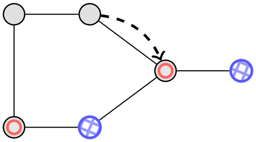

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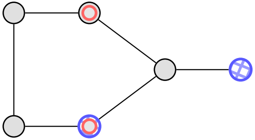

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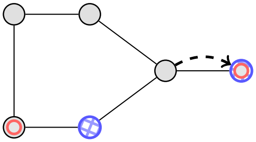

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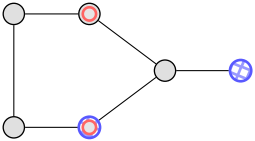

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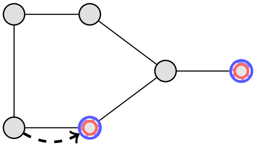

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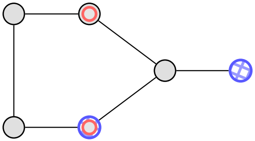

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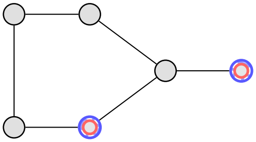

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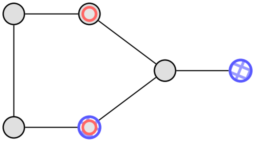

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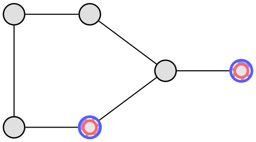

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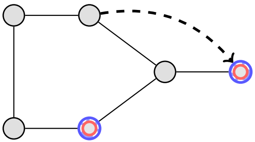

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<u>ISR Reachability - Token Jumping</u>

Input: A simple graph $G = (V, E)$, two independent sets I and J of G of same size.

Output: Yes if we can iteratively reach J from I using the Token Jumping rule, No otherwise.

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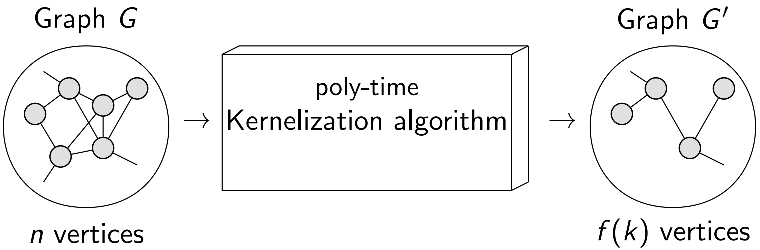
Parameterized hardness result (Mouawad, 2017)

TOKEN JUMPING is $W[1]$-hard (not FPT) when only parameterized by the number of tokens k.

Positive results: known kernels



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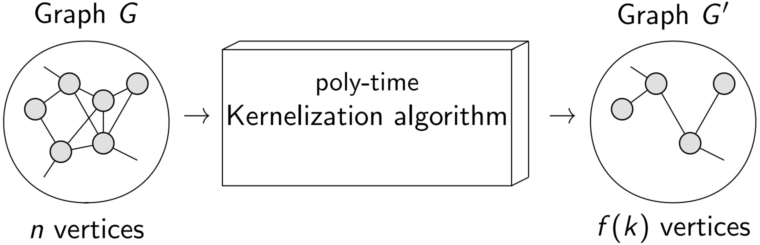

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- FPT on planar graphs and $K_{3,t}$-free graphs (Ito et al, 2014).
- Polynomial kernel for $K_{t,t}$-free graphs (Bousquet et al, 2017).
- Polynomial kernel on graphs of bounded degeneracy (Lokshtanov et al. 2018).

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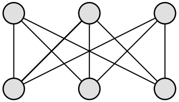

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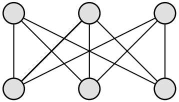

$K_{3,3}$ is not planar ($g \neq 0$)

$K_{3,3}$ embedded on the torus ($g = 1$)

In a nutshell, the genus g of a graph G is
the minimum number of handles required to draw G on a mug.

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Main result (Cranston, Mühlenthaler, P., 2024+)

Token Jumping parameterized by the genus g of the input graph and the number of tokens k admits a kernel of size $O((g+k)^2)$.
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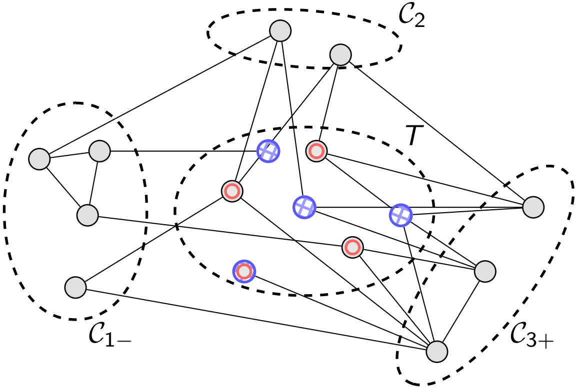
Positive kernelization results applied on graphs on surfaces:

Classes of graphs	Kernel size	For genus g
$K_{3,t}$-free (Ito et al, 14)	Ramsey$((2t+1)k, t+3)$	Ramsey$((8g+7)k, 4g+6)$
$K_{t,t}$-free (Bousquet et al, 17)	$O(f(t)\cdot k^{t\cdot 3^t})$	$O(h(g)\cdot k^{(4g+3)\cdot 3^{4g+3}})$
d-degenerate (Lokshtanov et al, 18)	$(2d+1)(2d+1)!(2k-1)^{2d+1}$	$(2H(g)-1)(2H(g)-1)!(2k-1)^{2H(g)-1}$
all graphs (This presentation!)	$O((g+k)^2)$	-

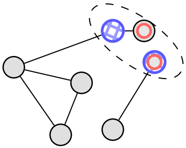
First step: Partition

- T: vertices containing the independent sets
- $\mathcal{C}_{1-}$: vertices neighboring at most one element of T
- $\mathcal{C}_2$: vertices neighboring exactly two elements of T
- $\mathcal{C}_{3+}$: vertices neighboring at least three elements of T



$\mathcal{C}_{1-}$ and $\mathcal{C}_{3+}$: easily bounded

Heawood's number $H(g) = \left\lfloor (7 + \sqrt{1 + 48g})/2 \right\rfloor$ is the maximum number of colors required to properly color a graph of genus g.
If $|\mathcal{C}_{1-}| \geq H(g) \cdot k$, the instance is YES. So we can assume



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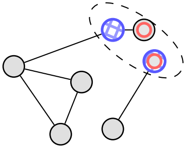

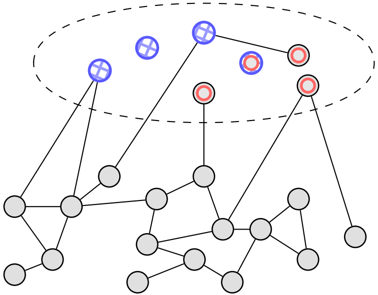

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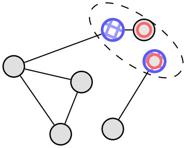

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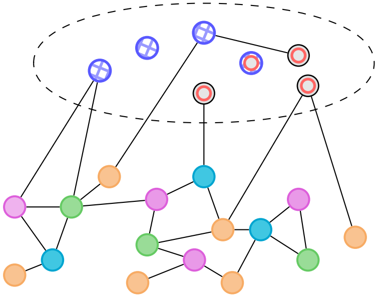

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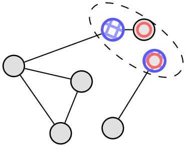

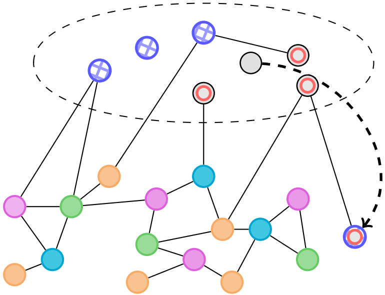

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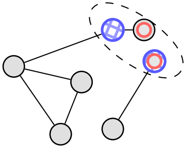

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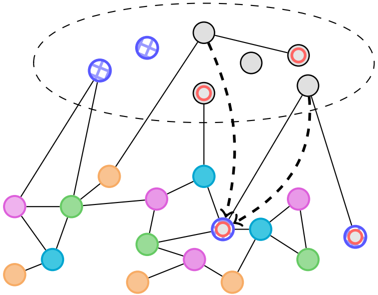

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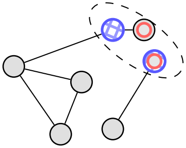

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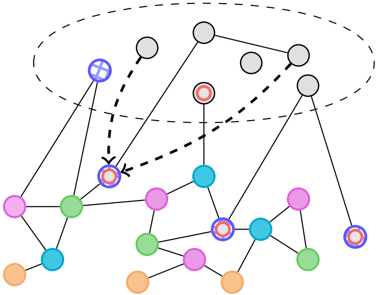

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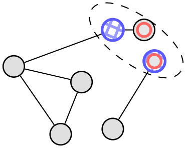

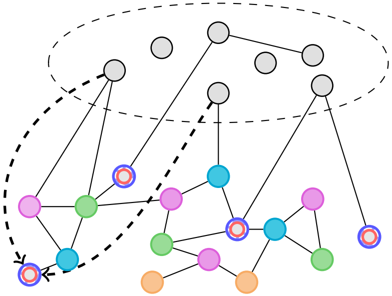

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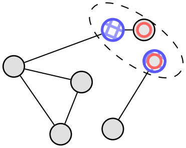

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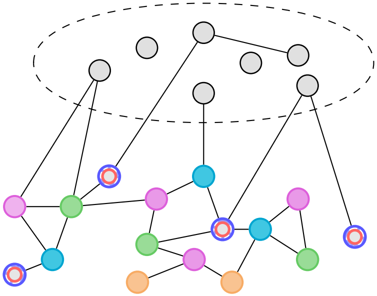

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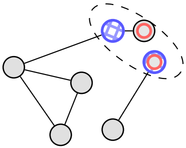
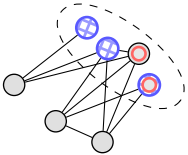


Theorem (Bouchet, 1978)

A graph of genus g cannot have any $K_{3,4g+3}$ as a subgraph.



Using an auxillary graph, we can use Euler's formula to get

$$|\mathcal{C}_{3+}| \leq 16g^2 + 16gk + 8k.$$

$\mathcal{C}_2$: not clear yet

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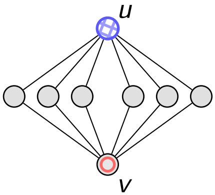

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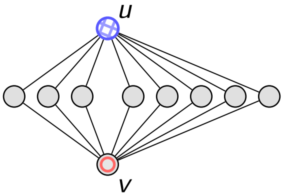

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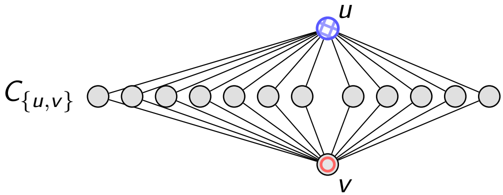

$\mathcal{C}_2$: not clear yet

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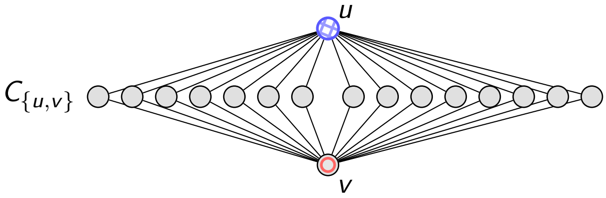

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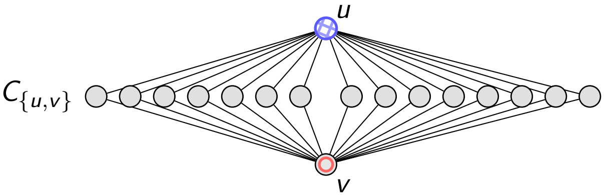

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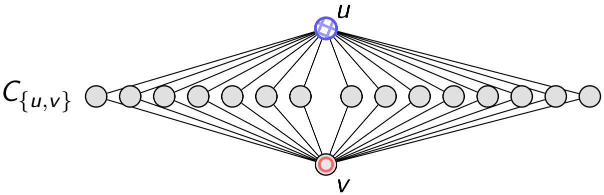

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By Euler's formula, the number of non-empty projection classes is at most $6k + 6g$.

We will show that if any $C_{\{u,v\}}$ is bigger than $8g + 4k$, the problem is solved.

Planar zones

Theorem (Malnič and Mohar, 1992)

The maximum number of non-homotopic internally disjoint u, v-paths on any graph of genus g is $\max(1, 4g)$.

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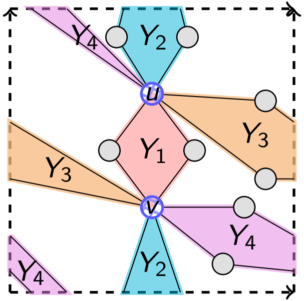
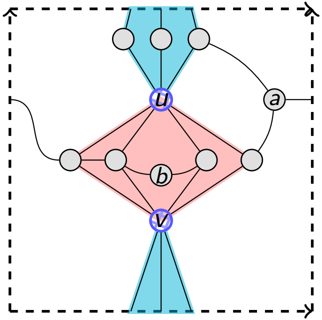
Hence, paths between u and v in $C_{\{u,v\}}$ divide the surface in at most $4g$ **planar zones**.



Four zones for $C_{\{u,v\}}$ on a torus.

Anatomy of the zone



Each zone has two **outer** vertices and some **inner** vertices.

Inner vertices form induced linear forests in $C_{\{u,v\}}$ whose independent sets are large and easy to find.

- Vertices outside a zone <u>cannot</u> be adjacent to inner vertices of $C_{\{u,v\}}$.
- Vertices inside a zone can only be adjacent to <u>two</u> vertices of $C_{\{u,v\}}$.

Problem solved

$$\begin{aligned} C_{\{u,v\}} \text{ is large } (8g + 4k) \quad &\implies \quad \geq 4k \quad \text{inner vertices} \\ &\implies \quad \geq 4k \quad \text{size linear forest} \\ &\implies \quad 2k \quad \text{size independent set } T_{\{u,v\}} \text{ in } C_{\{u,v\}} \end{aligned}$$

Recall each token of I is adjacent to at most two inner vertices of $C_{\{u,v\}}$.
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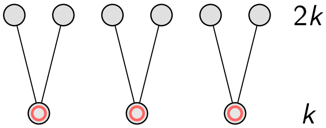

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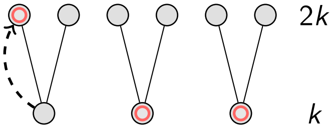

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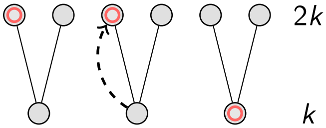

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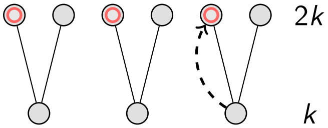

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Problem solved... or is it?

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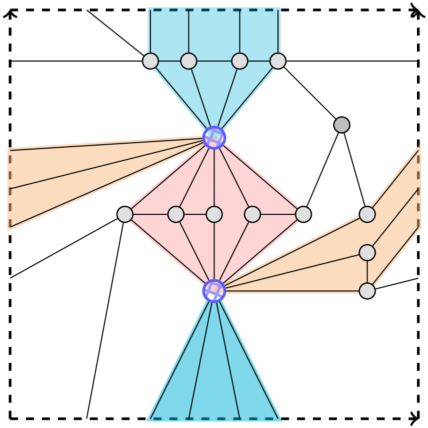
Problem: knowing the genus of the graph or a crossing-free drawing, is hard.

We will find that large linear forest without any information on the genus.

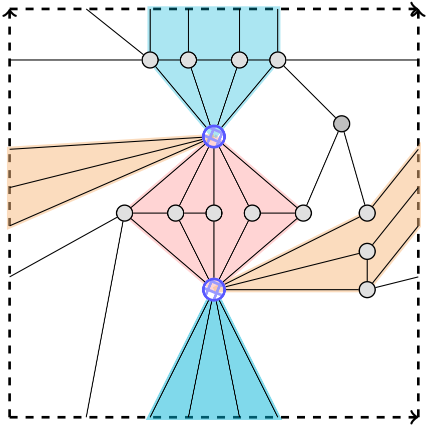
The algorithm

1 $Z := C_{\{u,v\}}$



The algorithm

$Z := C_{\{u,v\}}$
for $v \in V - (C_{\{u,v\}} \cup Y)$ **do**
  if *v has at least 3 neighbors in $C_{\{u,v\}}$* **then**
    $Z \leftarrow Z - N(v)$



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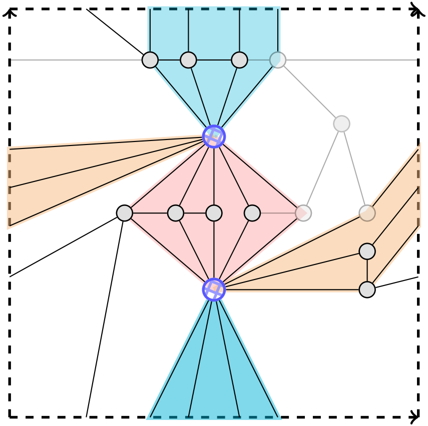

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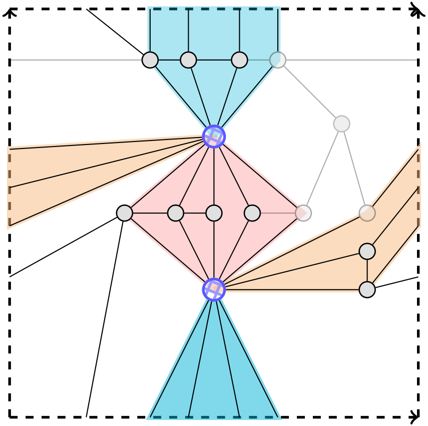

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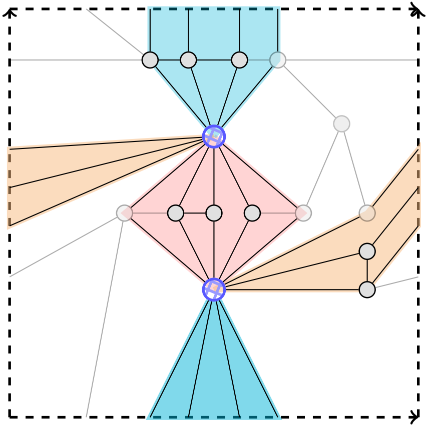

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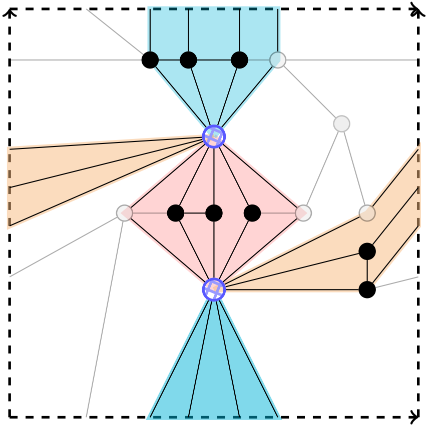
Remove arbitrarily one vertex from each cycle in $G[Z]$
return Z



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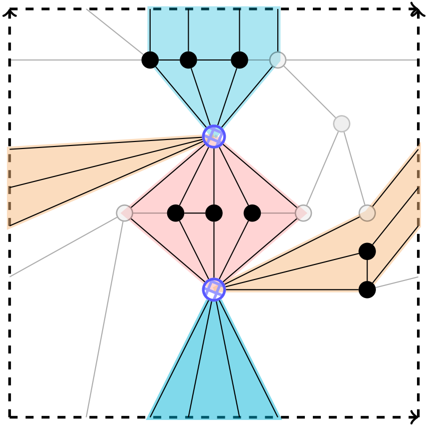

This procedure outputs a linear forest of size at least equal to the number of inner vertices, without any information on the genus.

Conclusion

We give a kernelization algorithm with quadratic size $O((g+k)^2)$ for Token Jumping.

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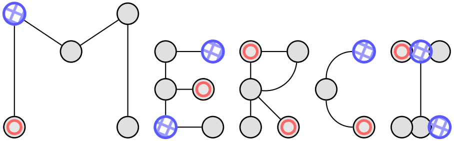

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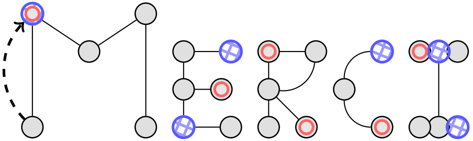

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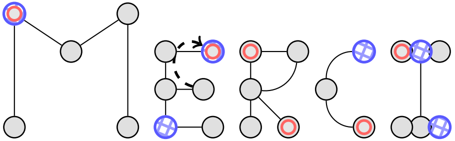

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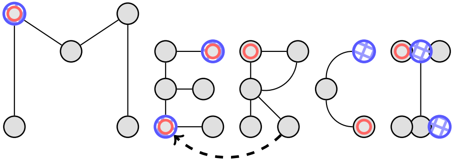

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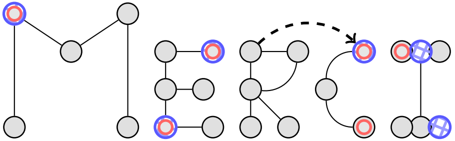

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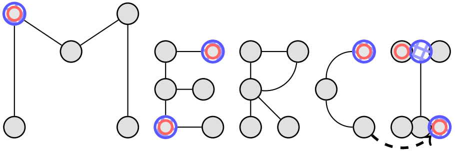

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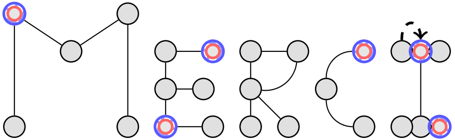

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